Light-front model of the kaon electromagnetic current *

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The electromagnetic form factor is extracted from both components of the electromagnetic current: J^+ and J^- with a pseudo-scalar coupling of the quarks to the kaon. In the case of J^+ there is no pair term contribution in the Drell-Yan frame ($q^+ = 0$). However, J^- component of the electromagnetic current the pair term contribution is different from zero and is necessary include it to preserve the rotational symmetry of the current.

Light-cone formalisms with model wave-functions have been applied in the literature [1,2] to study the pseudoscalar mesons [3–10]. In [8], the form factors in a pseudo-scalar coupling model are extracted using the components $J^+ = J^0 + J^3$ and $J^- = J^0 - J^3$ of the electromagnetic current, in the Drell-Yan frame ($q^+ = 0$). The pair term contribution is present in J^-, but not in J^+, where the contribution of the pair term vanishes. The pion J^+ current, with only light-cone valence wave function results to be equal to the covariant calculation, because the pair term is zero. In the case of vector particles, even the J^+ electromagnetic current has contribution from pair terms in order to respect the full covariance [11–13]. In order to satisfy the angular condition for spin one particles, it is necessary to consider pair term in the electromagnetic current J^+ (see the Ref.[11] and references therein). Besides the valence contribution to the J^- current, the pair term is necessary for both, pseudoscalar and vector particles to keep the rotational symmetry properties of the current in the light-front formalism. In order to extract the electromagnetic form factor for the kaon, the components J^+ and J^- of the electromagnetic current are used. The $J^{(\mu=\pm)}$ components of the electromagnetic current for the kaon have contribution, from the quark (q) and the antiquark ($\bar{q}$), given by

$$\begin{aligned} J^\mu_q(q^2) &= \imath e_q g^2 N_c \int \frac{d^4k}{(2\pi)^4} Tr[S(k-m_{\bar{q}})\gamma^5 S(P'-k-m_q)\gamma^\mu S(P-k-m_q)\gamma^5 \Lambda(k,P')\Lambda(k,P)] \ , \\ J^\mu_{\bar{q}}(q^2) &= q \leftrightarrow \bar{q} \ \text{ in } \ J^\mu(q^2) \ , \end{aligned} \tag{1}$$

where the number of colors is $N_c = 3$, g is the coupling constant and e_q ($e_{\bar{q}}$) is the quark (anti-quark) charge. The Breit frame is utilized, where the momentum transfer is $q^2 = -(\vec{q}_\perp)^2$, $P^0 = P'^0$ and $\vec{P}'_\perp = -\vec{P}_\perp = \frac{\vec{q_\perp}}{2}$. The function $\Lambda(k,p) = N/((p-k)^2 - M_R^2 + \imath\epsilon)$ is used in order to regulate the divergent integral, where M_R is the regulator mass and $m_{q(\bar{q})}$ is the quark (anti-quark) mass. The function $S(P)$ is the fermion propagator. The light-front coordinates are defined as $k^+ = k^0 + k^3$, $k^- = k^0 - k^3$, $k_\perp = (k^1, k^2)$. In the following, the method used in the calculation of the pair term, is the one developed in Ref.[12] for a composite boson bound state and in the study of the Ward-Takahashi identity in the light-front formalism [14]. The

*To be published by World Scientific in the proceedings of the "VIII International Workshop on Hadron Physics,(HADRONS 2002)", Bento Goncalves, RS, Brazil, April 14-19, 2002.

contribution of the pair term for J^+ and J^- components of the electromagnetic current comes from the matrix elements proportional to k^- in both cases (antiquark and quark on-shell).

In order to extract the form factor for the kaon, $F_{K^+}(q^2)$, we used both J^+ and J^- components of the electromagnetic current.

One can verify that only the on-shell pole $\bar{k}^- = (f_1 - \imath\epsilon)/k^+$ contribute to the k^- integration in the interval $0 < k^+ < P^+$:

$$\begin{aligned}
F^+_{\bar{q}}(q^2) &= e_q \frac{N^2 g^2 N_c}{P^+} \int \frac{d^2k_\perp dk^+}{2\ (2\pi)^3} \frac{-4(\bar{k}^- k^{+2} - k^+ k_\perp^2 - 2\bar{k}^- k^+ P^+ + 2k_\perp P^+ \bar{k}^- P^+ - k^+ q/4)}{k^+(P^+ - k^+)^2 (P'^+ - k^+)^2 (P^- - \bar{k}^- - \frac{f_2 - \imath\epsilon}{P^+ - k^+})} \\
&\quad \frac{-4k^+ m_q^2 + 8(k^+ - P^+) m_q m_{\bar{q}} \quad \theta(P'^+ - k^+)\theta(k^+ - P^+)}{(P'^- - \bar{k}^- - \frac{f_3 - \imath\epsilon}{P'^+ - k^+})(P^- - \bar{k}^- - \frac{f_4 - \imath\epsilon}{P^+ - k^+})(P'^- - \bar{k}^- - \frac{f_5 - \imath\epsilon}{P'^+ - k^+})} \ . \qquad (2) \\
F^+_q(q^2) &= [\ q \leftrightarrow \bar{q} \ \text{ in } F^+_{\bar{q}}(q^2)\ ] \ , \qquad (3)
\end{aligned}$$

where $f_1 = k_\perp^2 + m_{\bar{q}}^2$, $f_2 = (P-k)_\perp^2 + m_q^2$, $f_3 = (P'-k)_\perp^2 + m_q^2$, $f_4 = (P-k)_\perp^2 + M_R^2$, $f_5 = (P'-k)_\perp^2 + M_R^2$.

In the case the quark is on-shell, the pole contribution is $\bar{k}^- = (f_6 - \imath\epsilon)/k^+$ and $f_6 = k_\perp^2 + m_q^2$.

The light-front wave function for the kaon appears after the k^- integration

$$\Phi^i_{\bar{q}}(x, k_\perp) = \frac{1}{(1-x)^2} \frac{N}{(m^2_{K^+} - M_0^2)(m^2_{K^+} - M_R^2)} \ , \qquad (4)$$

where $x = k^+/P^+$. The function M_R^2 is

$$M_R^2 = M^2(m_{\bar{q}}, M_R) = \frac{k_\perp^2 + m_{\bar{q}}^2}{x} + \frac{(P-k)_\perp^2 + M_R^2}{1-x} - P_\perp^2 \quad , \qquad (\bar{q} \leftrightarrow q) \ . \qquad (5)$$

The free quark mass square is given by $M_0^2 = M^2(m_{\bar{q}}, m_{\bar{q}})$, $(\bar{q} \leftrightarrow q)$. For the final wave-functions ($\Phi^f_{\bar{q}}$ and Φ^f_q) is necessary exchange $P \leftrightarrow P'$.

The expression obtained for the electromagnetic form factor in terms of the wave function initial ($\Phi^i_{\bar{q}}$) and final ($\Phi^f_{\bar{q}}$) is

$$\begin{aligned}
F^+_{\bar{q}}(q^2) &= e_q \frac{N^2 g^2 N_c}{P^+} \int \frac{d^2k_\perp dx}{2(2\pi)^3 x} \Big[-4\Big(f_1 x P^+ - x P^+ k_\perp^2 - 2 f_1 P^+ + 2k_\perp^2 P^+ + \frac{f_1 P^+}{x} - \frac{x P^+ q^2}{4} \Big) \\
&\quad - \frac{4 f_1 P^+}{x} + 8P^+(x-1) m_q m_{\bar{q}} - 4xP^+ m_q^2 \Big] \theta(x)\theta(1-x)\ \Phi^{*f}_{\bar{q}}(x, k_\perp) \Phi^i_{\bar{q}}(x, k_\perp) \ , \qquad (6) \\
F^+_q(q^2) &= [\ q \ \leftrightarrow \ \bar{q} \text{ in } F^+_{\bar{q}}(q^2)\ ] \ . \qquad (7)
\end{aligned}$$

The final expression for the electromagnetic form factor obtained with J^+ is the sum of two contributions from the quark and the antiquark currents:

$$F^+_{K^+}(q^2) = F^+_q(q^2) + F^+_{\bar{q}}(q^2) \ , \qquad (8)$$

where the normalization is given by $F^+_{K^+}(0) = 1$. This expression is free of the pair term contributions, due the fact the pair term contributions for the J^+ component of the electromagnetic current is zero in the interval (II) [8]. The calculation of the kaon electromagnetic form factor in the light-front with J^+, without pair term, give the same result as the covariant one (see Fig.1).

The contribution to the electromagnetic form factor obtained with J^- from the interval (I) ($0 < k^+ < P^+$) is given by

$$\begin{aligned}
F^{-(I)}_q(q^2) &= 2\imath e_q \frac{N^2 g^2 N_c}{2P^+} \int \frac{d^2k_\perp dk^+ dk^-}{(2\pi)^4} \frac{-4(k^{-2}k^+ - k^- k_\perp^2 - 2k^- k^+ P^+ + 2k_\perp^2 P^+ + k^+ P^{+2}}{k^+(P'^+ - k^+)^2 (k^- - \frac{f_6 - \imath\epsilon}{k^+})} \times \\
&\quad \frac{-k^- q^2/4) - 4k^{-2} m_{\bar{q}}^2 + 8(k^- - P^+) m_q m_{\bar{q}}}{(P^- - k^- - \frac{f_2 - \imath\epsilon}{P^+ - k^+})(P'^- - k^- - \frac{f_3 - \imath\epsilon}{P'^+ - k^+})(P^- - k^- - \frac{f_4 - \imath\epsilon}{P^+ - k^+})(P'^- - k^- - \frac{f_5 - \imath\epsilon}{P'^+ - k^+})} \qquad (9)
\end{aligned}$$

where $f_2 = (P-k)_\perp^2 + m_q^2$ and $k^- = \dfrac{k_\perp^2 + m_q^2}{k^+} = f_6/k^+$. The second contribution comes from the antiquark current:

$$F_{\bar{q}}^{-(I)}(q^2) = -2\imath e_{\bar{q}} \frac{Ng^2N_c}{2P^+} \int \frac{d^2k_\perp dk^+ dk^-}{2\ (2\pi)^4} \frac{-4(k^{-2}k^+ - k^-k_\perp^2 - 2k^-k^+P^+ + 2k_\perp^2P^+ + k^+P^{+2}}{k^+(P'^+ - k^+)^2(k^- - \frac{f_1 - \imath\epsilon}{k^+})} \times$$
$$\frac{-k^-q^2/4) - 4k^{-2}m_q^2 + 8(k^+ - P^+)m_q m_{\bar{q}}}{(P^- - k^- - \frac{f_2 - \imath\epsilon}{P^+ - k^+})(P'^- - k^- - \frac{f_3 - \imath\epsilon}{P'^+ - k^+})(P^- - k^- - \frac{f_4 - \imath\epsilon}{P^+ - k^+})(P'^- - k^- - \frac{f_5 - \imath\epsilon}{P'^+ - k^+})} \quad (10)$$

where $k^- = \dfrac{k_\perp + m_{\bar{q}}^2}{k^+} = f_1/k^+$.

After the integration in k^- the electromagnetic form factors are

$$F_{\bar{q}}^{-(I)}(q^2) = e_q \frac{N^2g^2N_c}{P^+} \int \frac{d^2k_\perp dx}{2(2\pi)^3 x} \Big[-4\Big(\frac{f_1^2}{xP^+} - \frac{f_1k_\perp^2}{xP^+} - 2f_1P^+ + 2k_\perp^2P^+ + xP^{+3} - \frac{f_1q^2}{4xP^+}\Big)$$
$$-\frac{4f_1m_q^2}{xP^+} + 8(\frac{f_1}{xP^+} - P^+)m_q m_{\bar{q}}\Big] \theta(x)\theta(1-x)\ \Phi_{\bar{q}}^{*f}(x,k_\perp)\Phi_{\bar{q}}^{i}(x,k_\perp)\ . \quad (11)$$

$$F_q^{-(I)}(q^2) = [\ q \leftrightarrow \bar{q} \text{ in } F_{\bar{q}}^{-(I)}(q^2)\ ]\ . \quad (12)$$

When using J^- to extract the electromagnetic form factor, besides the contribution of the interval (I), the pair term contributes to the electromagnetic form factor in the interval (II) ($P^+ < k^+ < P'^+$). The pair term contribution for the form factor, as shown in Refs.[8,9], is given by $F^{-(II)}(q^2)$. The final expression for the electromagnetic form factor for the kaon, extracted from J^- is

$$F_{K^+}^-(q^2) = [F_q^{-(I)}(q^2) + F_{\bar{q}}^{-(I)}(q^2) + F^{-(II)}(q^2)]\ , \quad (13)$$

which is normalized by the charge, $F_{K^+}^-(0) = 1$.

The parameters of the model are the constituent quark masses $m_q = m_u = 0.220$ GeV, $m_{\bar{q}} = m_{\bar{s}} = 0.419$ GeV, and the regulator mass $M_R = 0.946$ GeV, which are adjusted to fit the electromagnetic radius of the kaon. With these parameters, the calculated electromagnetic radius of the kaon is $< r_{k^+}^2 >= 0.354\ fm^2$, that is very close to the experimental radius $< r_{k^+}^2 >= 0.340\ fm^2$ [15]. The electromagnetic form factor is presented in Fig.1. Due to the fact that J^+ does not have the light-front pair term contribution, the electromagnetic form factor from Eqs.(2) and (3) results equal to the one obtained in a covariant calculation. In the case of J^-, the light-front calculation with Eqs. (11) and (12) is quite different from the covariant results, as shown in Fig.1. After the inclusion of the pair term in the form factor calculated with J^-, Eq. (13), there is a complete agreement between the light-front and the covariant calculations.

In conclusion, the J^+ and J^- components of the electromagnetic current for the kaon are obtained in the light-front and in the covariant formalisms, in a constituent quark model. In the case of J^-, the inclusion of the pair term is essential to obtain the agreement between the covariant and the light-front calculations of the kaon electromagnetic form factor.

Our thanks to the Brazilian agencies FAPESP and CNPq for financial support.

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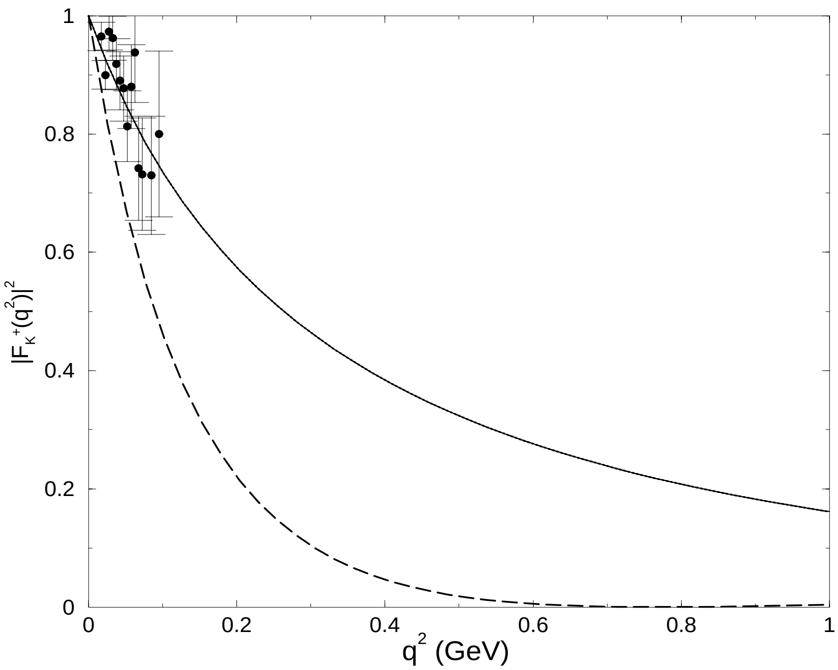

Figure 1. Kaon (K^+) form factor calculated within the covariant and the light-front formalisms with J^+ and J^-. The dashed line give the results from J^- without the light-front pair term. Adding the pair term to it, we obtain the result given by the solid line, which coincides with the light-front and covariant calculations with J^+. The experimental data comes from Refs. [15,16].

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